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APPLICATION OF TAGUCHI DESIGN OF EXPERIMENTS TECHNIQUES TO ESTIMATE COORDINATE MEASURING MACHINE UNCERTAINTY

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***Abstract.** The modern concept of measurement traceability involves the use of calibrated gauges and the expression of the measurement uncertainty. In the case of Coordinate Measuring Machines (CMM), there is controversy related to the most suitable method to determine the measurement uncertainty, due to the great number of variables affecting the measurement errors. This work presents a technique to determine the measurement uncertainty of CMM using Taguchi design of experiments. These techniques allow us to plan experiments where only a few runs are executed to investigate a greater number of variables. Five variables were studied in a Moving Bridge CMM when measuring a calibrated ball bar gauge. The standard uncertainty was estimated by the standard deviation of the measurement errors and estimates of the standard uncertainty associated to the machine variables were obtained through the components of variance, determined with the analysis of variance results. The results showed that the method is suitable to estimate the measurement uncertainty of complex measurement systems when the most significant uncertainty sources are of the type A.*

***Key-words:** Uncertainty analysis, Taguchi methods, Coordinate Measuring Machine*

1. INTRODUCTION

Coordinate Measuring Machines (CMM) uncertainty evaluation has long been discussed by researchers all around the world. This occurs because of the complexity of the CMM construction, resulting in a great number of variables affecting the measurement errors and the machine uncertainty. Moreover, the machine measures points in the work volume and the final uncertainty statement is a function of the determined point uncertainties.

There are three general approaches used to determine the measurement uncertainty of CMMs, comparison methods, performance tests methods and mathematical modeling. Amongst these approaches, the performance tests are more interesting to the CMMs users because of the reduced cost involved in its use. In these methods a calibrated gauge is measured in some previously defined positions in work volume as presented by international standards like American ANSI/ASME B89.4.1 and German VDI/VDE 2617 (Bosch, 1995).

The standard methods are used to facilitate CMMs commerce and the uncertainty estimate obtained is limited to compare different machines. An existent gap in these methods is the operational variables lack of control like position along the axes and orientation in work volume when measuring an object. Although these variables may be investigated, no quantitative conclusion is obtained about its effect. This can be obtained since statistical methods were applied (Piratelli-Filho, 1997).

Techniques of statistical design of experiments makes the variables effect over the measurement errors investigation and determination possible. These may be executed by machines variables controlled change and thus determining their effects over the measurement errors when measuring a calibrated gauge through a statistical analysis of variance [Montgomery, 1991].

Amongst the several experimental designs related in literature, the Taguchi experimental designs are suitable to use in problems where there are many variables to investigate and it is necessary to reduce drastically the total number of experimental runs. Taguchi arrays are equivalent to fractional factorial designs, but the codification applied by Taguchi facilitates the application by the inexperienced user [Vivacqua, 1995].

This work proposes a method to estimate the measurement uncertainty of CMMs through the use of Taguchi design of experiments. To investigate five CMM variable effects, the L9 Taguchi array are employed. A calibrated ball bar gauge is used to estimate the measurement errors in each run of the experimental design. The analysis of variance is used to determine the variables that most influence the measurement errors and each experimental variable components of variance. The CMM combined standard uncertainty is determined through the measurement errors standard deviation and the standard uncertainty associated to experimental variables are determined by the components of variance.

2. TAGUCHI EXPERIMENTAL TECHNIQUES

Taguchi design of experiments are considered a group of experimental techniques used to improve the quality of products in industry. Their central idea concerns the product functional characteristics variability reduction. The variability sources are investigated through running a series of experimental tests using planned experiments (Vivacqua, 1995).

The planned experiments starts with the experimental variables selection. Thus, the control variables and their levels are selected to investigate the desirable response variable behavior. The next step concerns the choice of Taguchi experimental array, in order to conduct the experiment and minimize the total number of variable level combinations (runs). There are a lot of experimental arrays proposed by Taguchi, presented in a catalog in order to simplify the selection and facilitate the use by the inexperienced user.

Some Taguchi experimental arrays are classical fractional factorial experiments. The proposed catalog have several arrays with two to five level variables and the most important are 2 and 3 level fractional factorials. These arrays are minimum fractions of the complete factorial designs, in order to minimize the total number of runs. When calibrating an instrument, one run consists of measuring a standard artifact after adjust the controlled variables in levels predicted by the line of experimental array [Kacker et alii, 1991].

2.1 Taguchi L9 array

Amongst the Taguchi proposed arrays, the most used are the two and three levels called L8, L9 and so on. These arrays may be considered as generalized Latin Squares by the design of experiments literature and the letter "L" was adopted by Taguchi to express these arrays. The corresponding number is the total number of runs in each array (Montgomery, 1991).

The L9 array is a 3^{4-2} fractional factorial design with 9 runs, obtained as 1/9 fraction of the 3^4 factorial design with 81 runs. This construction permits to investigate up to four control variables in an experiment executing only nine runs. This array is showed in Table 1, where A, B, C and D are the control variables and the numbers 1, 2 and 3 are their levels. The runs are a combination of the variables levels and the first run must be executed controlling the variables A, B, C and D at level 1.

Table 1 - Array L9 proposed by Taguchi.

Run	A	B	C	D
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

The experimentation sequence must be a random to reduce the influence of some external variables not controlled, like the laboratory room temperature. A variable effect is the change in response variable introduced by its levels controlled modification. The array high degree of fractionation results a complex alias structure, where variables A to D effects are mixed with two or more variable interaction effects (Piratelli-Filho, 1997).

A fifth variable can be investigated using this array since blocs are attributed to control it. These blocs are introduced repeating each L9 run only changing the fifth variable (E) in its "e" values or levels. If the interactions effects may be ignored, a mathematical model may be established as showed in Eq. (1) representing the response variable value y_{ijloqw} as a function of the variables A to E effects at its levels (τ_i, β_j, γ_l, δ_o, η_q) in relation to the total mean value of response variable μ. The value ε_{ijloqw} represents the models residuals, e.g., the difference between the experimental and predicted response variable value. The indices i, j, l, o and q represents the currents levels of the variables A, B, C, D and E, respectively, and w is the replicates number obtained at each different variables levels combination. The indices values used in the proposed method are indicated in Eq. (1).

$$y_{ijloqw} = \mu + \tau_i + \beta_j + \gamma_l + \delta_o + \eta_q + \varepsilon_{ijloqw} \begin{cases} i = 1,2,3 \\ j = 1,2,3 \\ l = 1,2,3 \\ o = 1,2,3 \\ q = 1,2,\Lambda,6 \\ w = 1,2,3 \end{cases} \quad (1)$$

2.2 Analysis of experimental design

The experimental results analysis is executed through statistical analysis of variance. In these methods, the results total variation is expressed by its variance and determined by Eq. (2). The variance estimation associated to control variables are obtained by the decomposition of the total sum of squares (SST) in the sum of squares attributed to variables A to E, SSA to SSE, as showed in Eq. (3) to (7). It is important to point out that SST is obtained by adding SSA, SSB, SSC, SSD and SSE, and thus the residual sum of squares SSR are determined by difference. In these expressions, the indices represented by dots implies sums of all response variable values for all values of these omitted indices. For example, the term y...... represents the addition of all the response values y for all indices i to w values showed in Eq. (1) and the quadratic operation is executed only after this calculation is performed.

$$SST = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{l=1}^{c}\sum_{o=1}^{d}\sum_{q=1}^{e}\sum_{w=1}^{r} y_{ijloqs}^2 - \frac{y_{......}^2}{a^2 \cdot e \cdot r} \quad (2)$$

$$SSA = \sum_{i=1}^{a} \frac{y_{i.....}^2}{a \cdot e \cdot r} - \frac{y_{......}^2}{a^2 \cdot e \cdot r} \quad (3)$$

$$SSB = \sum_{j=1}^{b} \frac{y_{.j....}^2}{b \cdot e \cdot r} - \frac{y_{......}^2}{a^2 \cdot e \cdot r} \quad (4)$$

$$SSC = \sum_{l=1}^{c} \frac{y_{..l...}^2}{c \cdot e \cdot r} - \frac{y_{......}^2}{a^2 \cdot e \cdot r} \quad (5)$$

$$SSD = \sum_{o=1}^{d} \frac{y_{...o..}^2}{d \cdot e \cdot r} - \frac{y_{......}^2}{a^2 \cdot e \cdot r} \quad (6)$$

$$SSE = \sum_{q=1}^{e} \frac{y_{....q.}^2}{a^2 \cdot r} - \frac{y_{......}^2}{a^2 \cdot e \cdot r} \quad (7)$$

The variable variances and the residual variance are obtained by the mean squares (MS) dividing the respective sums of squares by its respective degrees of freedom (DF). The degrees of freedom are statistical measures of the variances independence degree and the variables degrees of freedom are determined by the difference between the total number of levels minus one degree of freedom lost when determining the mean. The total degrees of freedom is obtained through determining the number of total response variable values (N) minus one degree of freedom lost when determining the mean. The residual degrees of freedom is calculated by the total number of degrees of freedom minus the sum of all variables degrees of freedom. The Table 2 shows the analysis of variance table.

The most important variables influencing the response variable are determined by statistical Snedecor F test. In this test, the experimental variables F values are determined dividing the respective mean squares (MSA, MSB, MSC, MSD and MSE) by the residual mean squares (MSR). The calculated values are then compared with F values predicted by statistical F distribution, at 95% and at 99% confidence levels. When the F calculated is bigger than the standard F, the variable is considered to significantly influence the response variable at the respective confidence level and is indicated by asterisks (*) to 95% and (**) to 99% (Montgomery, 1991).

Table 2 - Analysis of variance table

SV	SS	DF	MS	F
A	SSA	a-1	MSA = SSA/(a-1)	FA = MSA/MSR
B	SSB	b-1	MSB = SSB/(b-1)	FB = MSB/MSR
C	SSC	c-1	MSC = SSC/(c-1)	FC = MSC/MSR
D	SSD	d-1	MSD = SSD/(d-1)	FD = MSD/MSR
E	SSE	e-1	MSE = SSE/(e-1)	FE = MSE/MSR
Residual	SSR	DFR	MSR = SSR/DFR	
Total	SST	N-1		

3. UNCERTAINTY ANALYSIS USING TAGUCHI ARRAYS

Accordingly to the recent ISO Guide to the Expression of Measurement Uncertainty (1993), the measurement uncertainty may be expressed by a standard deviation. When the measurement variable is a function of other variables, the measurement uncertainty value is called combined standard uncertainty and it is determined by the variables variance squared root. Each variable variance estimate may be used to determine the individual variable uncertainty estimate, called standard uncertainty and it is represented by a standard deviation (Bosch, 1995). When these individual uncertainties are determined through statistical analysis they are called type A and when they are estimated by other means, they are called type B.

The design of experiment techniques allow the control variables investigation over the response variable as the CMM measurement errors. By its nature, its application may stimulate the measurement errors variability since the CMM control variables are modified in levels that cover its entire range of permissible variation. Since these investigated variables are the most significant to promote the measurement errors variability, an estimate of the combined standard uncertainty may be obtained by the squared root of the total mean squares obtained in analysis of variance.

The variables mean squares determined in analysis of variance are only estimates of the standard deviations associated to variables. It must be proved that these mean squares may be decomposed in parts associated one to variable effects variance and the other to residual variance. The residual variance always affects any CMM measurement value and it may be associated to probe errors or other uninvestigated error sources that promote the variability in the results. This residual variance may be represented by σ^2 and is estimated by the expected residual mean squares, as showed in Eq. (13).

It must be noted that even if the controlled variables are not modified, there is some variability presented as measurement errors. Thus the mean squares determined in analysis of variance detects the residual variance estimate. This may be written like Eq. (8) to (12), where the variables A, B, C, D and E mean squares expected may be determined by the residual variance σ^2 added respectively to the variances σ_τ^2, σ_β^2, σ_γ^2, σ_δ^2 and σ_η^2 associated to the change of the variable levels.

$$E(MSA) = \sigma^2 + a \cdot e \cdot r \cdot \sigma_\tau^2 \quad (8)$$

$$E(MSB) = \sigma^2 + b \cdot e \cdot r \cdot \sigma_\beta^2 \quad (9)$$

$$E(MSC) = \sigma^2 + c \cdot e \cdot r \cdot \sigma_\gamma^2 \quad (10)$$

$$E(MSD) = \sigma^2 + d \cdot e \cdot r \cdot \sigma_\delta^2 \quad (11)$$

$$E(MSE) = \sigma^2 + a^2 \cdot r \cdot \sigma_\eta^2 \quad (12)$$

$$E(MSR) = \sigma^2 \quad (13)$$

These variable variances estimated are the variable levels change result and then may be calculated as a function of variable and residual mean squares using the Eq. (14) to (18). The denominator values depend on the variables levels adopted and the values in these expressions were calculated using the level values as expressed in Eq. (1).

The hypothesis admitted when using these equations is that the mean effects of changing the levels of the variable A, e.g. the difference between the measurement error value determined in each level of the variable A and the mean measurement error value of the machine, are independent and may be considered as a random variable presenting normal distribution with mean zero and variance σ_τ^2. These hypotheses may be established to variables B, C, D and E, leading to variances σ_β^2, σ_γ^2, σ_δ^2 and σ_η^2. In this case, the variance value squared root may then be considered as estimates of standard uncertainty associated to A to E control variables.

$$\hat{\sigma}_A^2 = \frac{MSA - MSR}{54} \quad (14)$$

$$\hat{\sigma}_B^2 = \frac{MSB - MSR}{54} \quad (15)$$

$$\hat{\sigma}_C^2 = \frac{MSC - MSR}{54} \quad (16)$$

$$\hat{\sigma}_D^2 = \frac{MSD - MSR}{54} \tag{17}$$

$$\hat{\sigma}_E^2 = \frac{MSE - MSR}{27} \tag{18}$$

The expanded uncertainty may be determined using a coverage coefficient that multiplies the standard or combined standard uncertainty (ISO Guide, 1993). In this way, the uncertainty value establishes a confidence interval that holds the measurement errors with a given probability. The normal distribution is generally adopted and the coefficient used at 95% confidence is 2.0. When the total degrees of freedom is smaller than 30, its recommended use the statistical t distribution to establish the coefficient (Montgomery, 1991).

4. EXPERIMENTAL TESTS

Experimental tests were conducted to check the ability of this design of experiment techniques in determining the CMM measurement uncertainty. The first step was the machine experimental variable selection amongst those which most influence the measurement errors to attribute to L9 array columns. After that, the experimental runs must be conducted in a random sequence.

A ball bar gauge was selected to determine the measurement errors. This gauge consists of two balls placed at the ends of a cylindrical bar and the distance between the ball centers must be calibrated before the measurement had beginning. This gauge great advantage is its construction low cost.

A Software was developed to determine the analysis of variance and the uncertainty values and thus simplify the calculations. The program was written in Turbo Pascal 6.0 and offers facility to the users.

4.1 Design of the experiment

Once the Taguchi L9 design was selected to conduct the experiments, five machine control variables were chosen. These variables were the position along the axes X, Y and Z, the ball bar length and the measurement orientation. The orientation was studied in six levels, while the others were studied in 3 levels each. These variables are associated to CMM operation and may be treated as operational variables.

The ball bar length was adopted as the distance between the balls centers and its bigger value must be chosen considering that it must have be positioned at three different locations along the X, Y and Z axes.

The positions along the X, Y and Z-axes must be chosen considering the bigger ball bar. Taking the ball bar at its length center, the three levels may be determined and are represented by points showed in Fig. 1. The white points represent the locations predicted by the Taguchi L9 array.

The orientations adopted must be sensible to machine twenty-one geometric errors. The Figure 2 shows the orientations recommended to run the experiment, like orientations along X and Y axes, at the plane diagonals XY, XZ and YZ and at the volume diagonal XYZ.

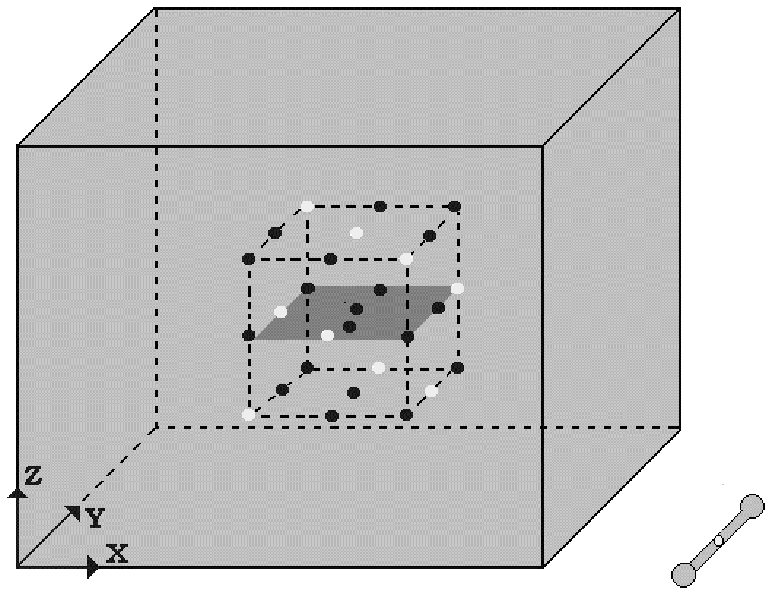

Figure 1 - Positions of measurement in CMM work volume

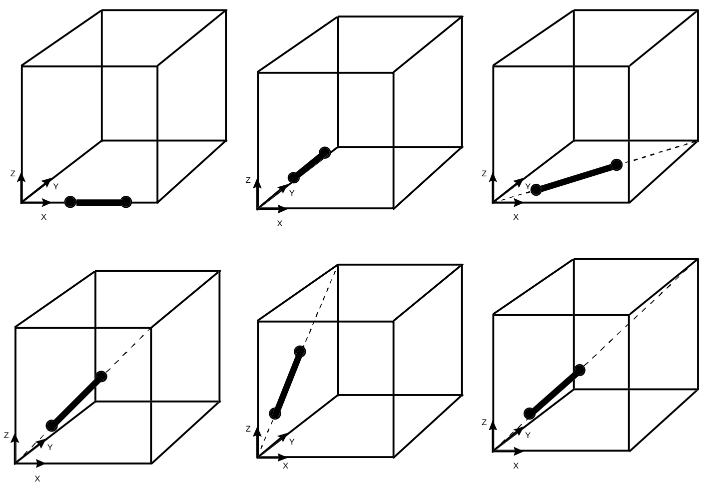

Figure 2 - Ball bar orientations in CMM work volume
array L9 and explanation of runs and variables proposed

4.2 Experimental Results

An experimental test was run on a Moving Bridge CMM at the University of São Paulo, São Carlos, Brazil. A touch trigger probe was used to measure the ball bar and seven equi-spaced points were touched over the spheres. The CMM software was used to determine the distance between the ball centers. The measurement errors were determined by calculating ball bar length measured minus the calibrated ones. Each measurement executed was repeated 3 times.

The variable levels adopted were the following: Orientation (variable E): X, Y, XY, XZ, YZ and XYZ; ball bar length (variable D): 131.3499, 197.4931 and 269.1468 mm; position along X axis (variable A): 146, 179 and 212 mm; Position along the Y axis (variable B): 146, 200 and 254 mm; position along the Z axis (variable C): 95, 130 and 165 mm.

The analysis of variance with measurement errors were determined and is showed in table 3. After having conducted the F test, it must be observed that the orientation and ball bar length variable influence the measurement errors at 99% confidence and the position along Z-axis influences at 95% confidence.

Table 3 - Analysis of variance - L9 experiment.

SV	SS	DF	MS	F
A	214.199	2	107.099	2.69
B	228.003	2	114.002	2.86
C	352.090	2	176.045	4.42 *
D	2104.104	2	1052.052	26.39 **
E	39060.348	5	7812.070	195.94 **
Residual	5900.621	148	39.869	
Total	47859.365	161		

The performance test proposed by ANSI/ASME B89.4.1 standard was executed measuring a 197.4931 mm ball bar according to eighteen positions and orientations established by this standard. The repeatability test was realized measuring a sphere positioned at the center of CMM work volume.

4.3 Uncertainty analysis

The CMM combined standard uncertainty, obtained with the measurement errors, was 17 µm. The expanded uncertainty was determined using k = 1.96 as a coverage factor, to 95% confidence level and Student distribution with 161 degrees of freedom. The B89 repeatability test showed a standard deviation of 16 µm, next to the combined standard uncertainty determined using the Taguchi L9 array.

The standard uncertainties associated to experimental variables position in X axis, position in Y axis, position in Z axis, ball bar length and orientation in work volume were respectively 1 µm, 1 µm, 2 µm, 4 µm and 17 µm. These results were confirmed through measuring ball bar gauges in some different levels of the variables studied.

5. CONCLUSION

The design of experiment techniques were efficient to estimate the CMM measurement uncertainty. Once the machine most important variables may be statistically studied, the controlled change of its levels had stimulated the measurement error variability. These methods may be recommended when the type A sources of uncertainty are the most important.

The use of Taguchi L9 array simplified the application of design of experiments and allowed to reduce the total number of runs in the calibration experiment. Besides, the uncertainty estimates obtained using Taguchi L9 array were close to the uncertainty values obtained by others techniques, like ANSI/ASME B89.4.1 and complementary tests applied.

The time consumed in application of the proposed technique was nearly equivalent to the time required to conduct the ANSI/ASME B89.4.1 performance and repeatability tests. But this time may be significantly reduced by automation of the experimental procedures involved in the ball bar measurement.

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